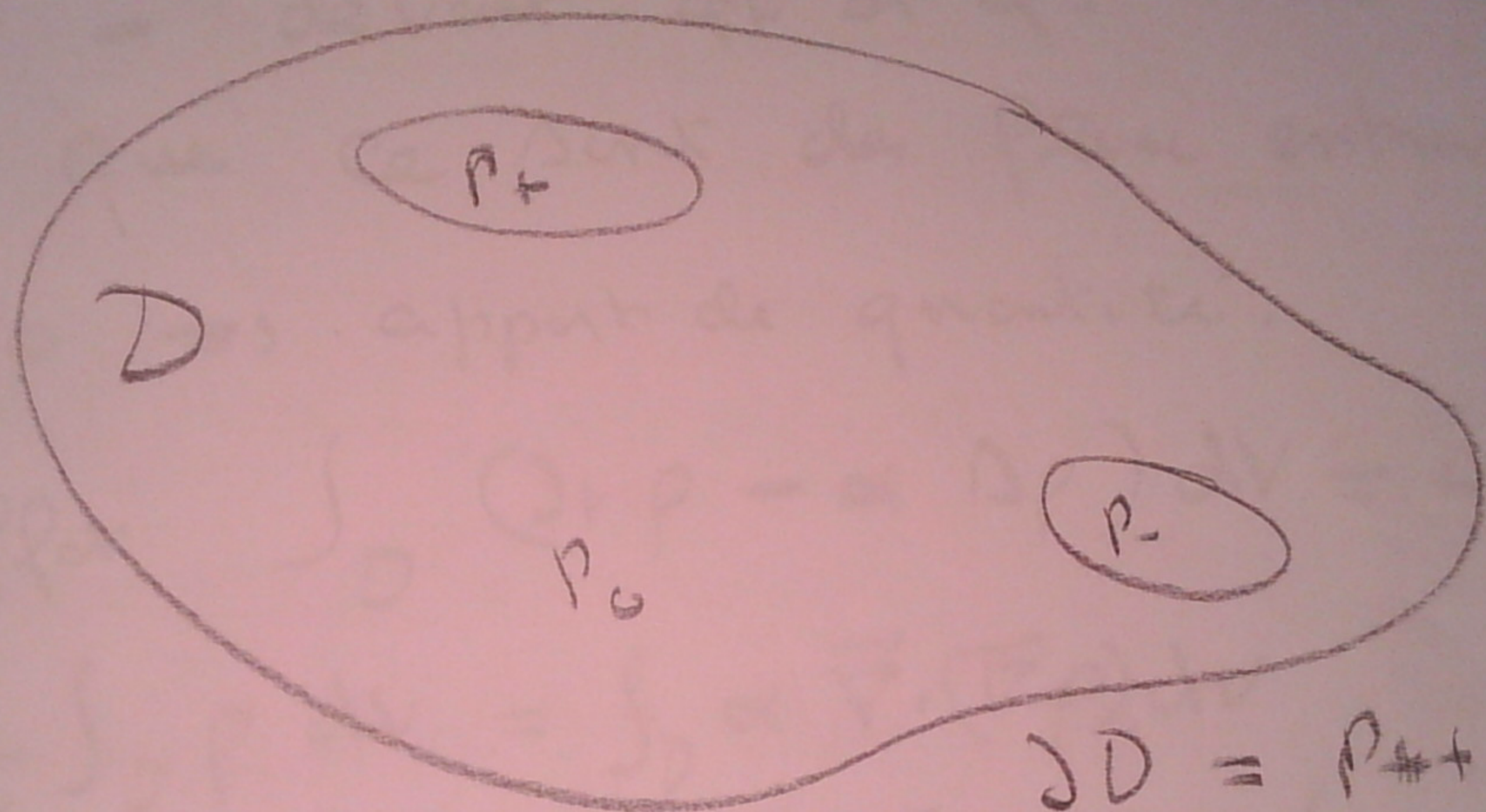


①



$$\partial D = \rho_+ + \rho_0 + \rho_-$$

$$(\Delta = \vec{\nabla} \cdot \vec{\nabla})$$

$$\partial_t \rho - \alpha \Delta \rho = 0 \text{ in } D$$

$$\rho(t=0, \vec{x}) = \rho_0(\vec{x}) \text{ in } D$$

$$-\alpha \vec{\nabla} \rho \cdot \vec{n} = -\phi_+ \text{ on } \rho_+$$

$$-\alpha \vec{\nabla} \rho \cdot \vec{n} = -\phi_- \text{ on } \rho_-$$

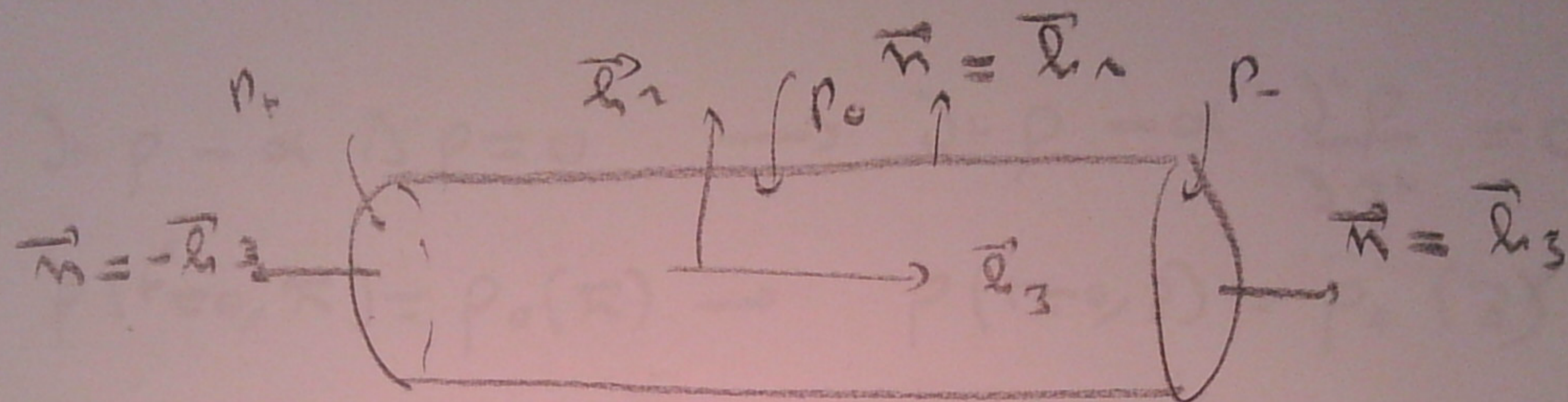
$$-\alpha \vec{\nabla} \rho \cdot \vec{n} = 0 \text{ on } \rho_0$$

② les '-' devant Φ_+ et Φ_- sont là
 parce que ce sont des flux entrant,
 $\Phi_+ > 0 \Rightarrow$ apport de quantité.

En effet $\int_D (Q + \rho - \alpha \Delta \rho) dv = 0$

$$\begin{aligned} \rightarrow \frac{d}{dt} \int_D \rho dv &= \int_D \alpha \nabla \cdot (\nabla \rho) dv \\ &= \int_{\Gamma_+} \alpha \nabla \rho \cdot \vec{n} ds + \int_{\Gamma_-} \alpha \nabla \rho \cdot \vec{n} ds \\ \frac{d}{dt} Q &= \Phi_+ (\int_{\Gamma_+} ds) + \Phi_- (\int_{\Gamma_-} ds) \end{aligned}$$

③



$$\begin{aligned} \vec{u} &= n \vec{u}_1 + 3 \vec{u}_3 \\ &\quad \cos \theta \vec{u}_1 + \sin \theta \vec{u}_3 \end{aligned} \quad \left. \vphantom{\begin{aligned} \vec{u} &= n \vec{u}_1 + 3 \vec{u}_3 \\ &\quad \cos \theta \vec{u}_1 + \sin \theta \vec{u}_3 \end{aligned}} \right\} p(t, \vec{u}) = p(t, n, \theta, 3)$$

on suppose que $p(t, n, \theta, 3) = p(t, 3)$

idem $p_0(n, \theta, 3) = p_0(3)$

et ϕ_+ , ϕ_- sont des constantes

$$D = \{ n \vec{u}_1 + 3 \vec{u}_3 / 0 \leq n \leq R, 0 \leq 3 \leq L \}$$

④

$$\partial_t \rho - \alpha \Delta \rho = 0 \quad \rightarrow \quad \partial_t \rho - \alpha \frac{\partial^2 \rho}{\partial z^2} = 0$$

$$\rho(t=0, x) = \rho_0(x) \quad \rightarrow \quad \rho(t=0, z) = \rho_0(z)$$

$$\vec{\nabla} \rho = \frac{\partial \rho(t, z)}{\partial z} \vec{e}_z \quad \Rightarrow$$

- $+ \alpha \vec{\nabla} \rho \cdot \vec{n} = 0 \text{ sur } P_0 \quad \text{vérifié!}$

- $+ \alpha \vec{\nabla} \rho \cdot \vec{n} = \phi_+ \text{ sur } P_+$

$$\rightarrow + \alpha \frac{\partial \rho(t, z=0)}{\partial z} = \phi_+$$

- $+ \alpha \vec{\nabla} \rho \cdot \vec{n} = \phi_- \text{ sur } P_-$

$$\rightarrow + \alpha \frac{\partial \rho(t, z=L)}{\partial z} = \phi_-$$

⑤ Au total

$$\partial_t \rho - \alpha \frac{\partial^2 \rho}{\partial z^2} = 0 \quad 0 < z < L$$

$$\rho(t=0, z) = \rho_0(z) \quad 0 < z < L$$

$$- \alpha \frac{\partial \rho}{\partial z}(t, z=0) = \phi_+$$

$$+ \alpha \frac{\partial \rho}{\partial z}(t, z=L) = \phi_-$$

Pb unidirectional / 1D

⑥ On retrouve le bilan global

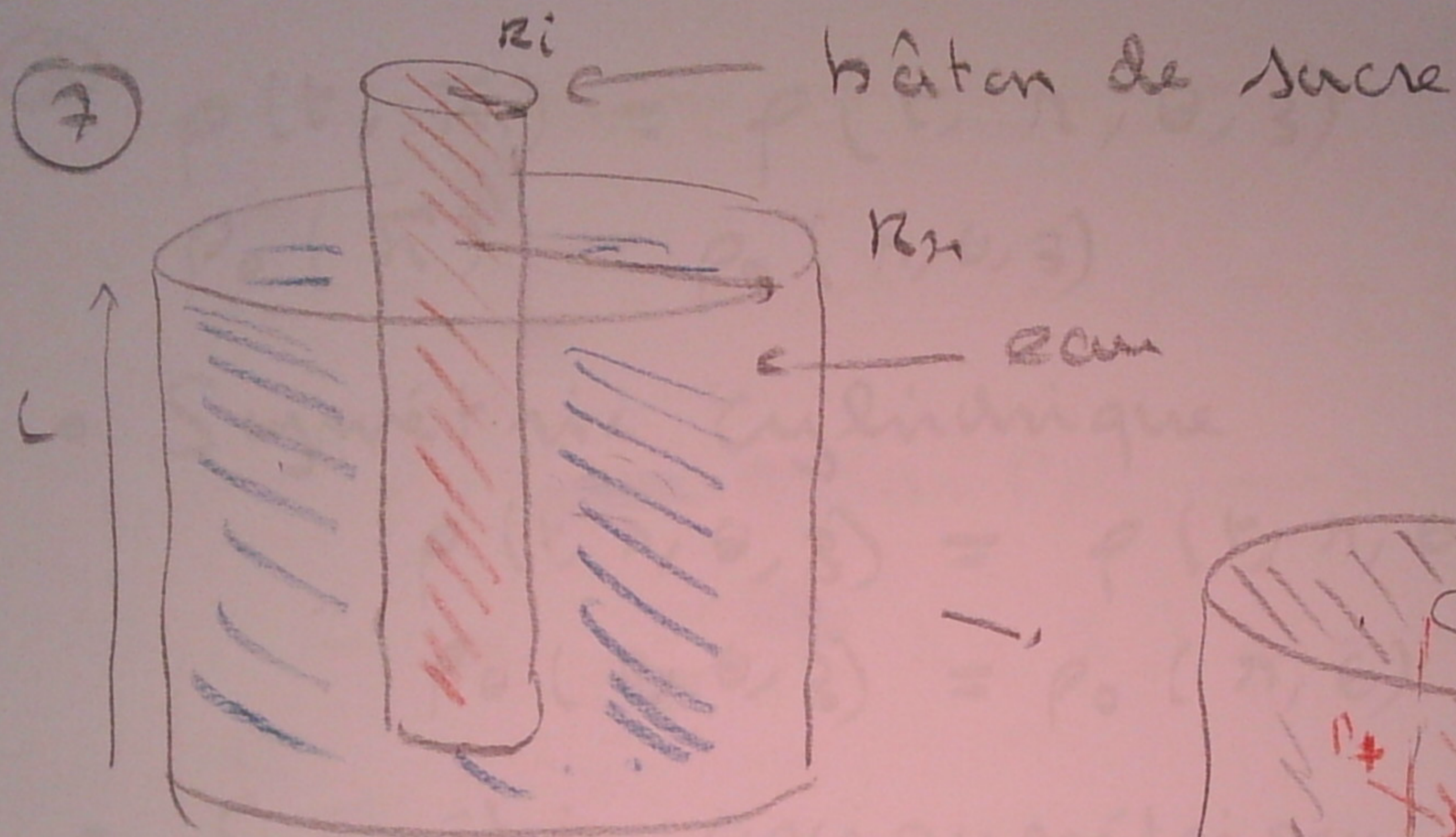
$$\int_0^L \left(\partial_t \rho - \alpha \frac{\partial^2 \rho}{\partial z^2} \right) dz = 0$$

$$\begin{aligned} \Rightarrow \frac{d}{dt} \int_0^L \rho dz &= \left[\alpha \frac{\partial \rho}{\partial z} \right]_0^L \\ &= \phi_+ + \phi_- \end{aligned}$$

Soit

$$\frac{d}{dt} \left(\pi R^2 \int_0^L \rho dz \right) = \pi R^2 \phi_+ + \pi R^2 \phi_-$$

$= Q$
 $\parallel$
 $\int_{p_+} ds$
 $\parallel$
 $\int_{p_-} ds$

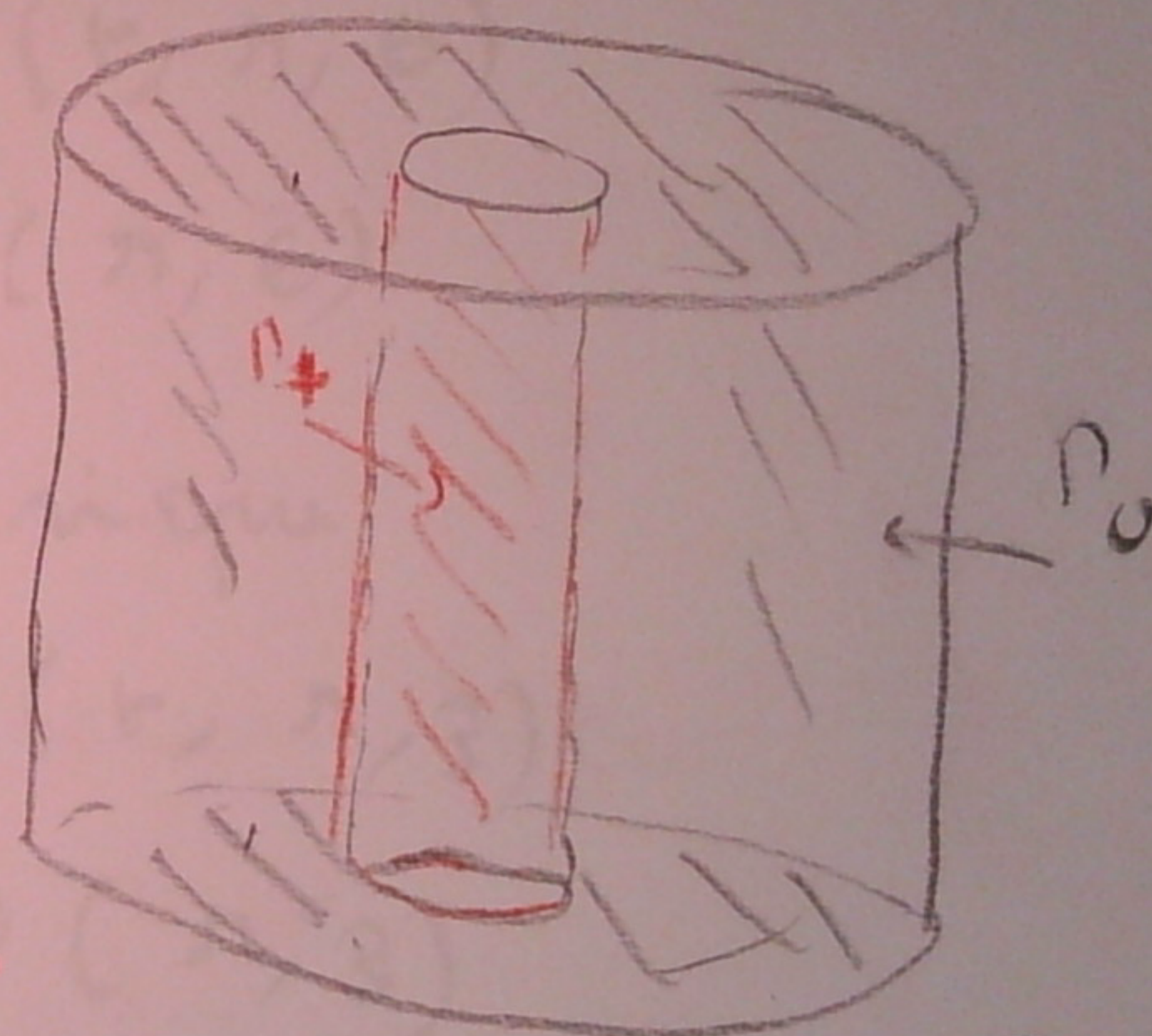


$$D = \left\{ \lambda \vec{e}_\lambda + z \vec{e}_z \mid 0 \leq z \leq L, R_i \leq \lambda \leq R_n \right\}$$

$$\partial D = \left\{ R_i \vec{e}_\lambda + z \vec{e}_z \mid 0 \leq z \leq L \right\} \cup$$

$$+ \left\{ R_n \vec{e}_\lambda + z \vec{e}_z \mid 0 \leq z \leq L \right\}$$

$$+ \left\{ \lambda \vec{e}_\lambda \mid R_i \leq \lambda \leq R_n \right\} + \left\{ \lambda \vec{e}_\lambda \mid R_i \leq \lambda \leq R_n \right\}$$



R_o

$$\textcircled{2} \quad \rho(t, \vec{r}) = \rho(t, r, \theta, z)$$

$$\rho_0(\vec{r}) = \rho_0(r, \theta, z)$$

• Symétrie cylindrique

$$\rho(t, r, \theta, z) = \rho(t, r, \theta)$$

$$\rho_0(r, \theta, z) = \rho_0(r, \theta)$$

• Symétrie axi-symétrique

$$\rho(t, r, \theta, z) = \rho(t, r, z)$$

$$\rho_0(r, \theta, z) = \rho_0(r, z)$$

⑨ Avec la 2 symétries $\begin{pmatrix} \rho(t, r) \\ \rho_0(r) \end{pmatrix}$

$$\partial_t \rho - \alpha \Delta \rho = 0 \rightarrow \partial_t \rho - \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \rho}{\partial r} \right) = 0$$

$$- \alpha \vec{\nabla} \rho \cdot \vec{M} = -\phi_+ \text{ sur } P_+ \rightarrow \quad R_i < r < R_n$$

$$+ \alpha \frac{\partial \rho}{\partial r}(t, r = R_i) = -\phi_+$$

$$- \alpha \vec{\nabla} \rho \cdot \vec{M} = 0 \text{ sur } P_0 \rightarrow$$

$\rightarrow$ implicite sur la demi base

$$- \alpha \frac{\partial \rho}{\partial r}(t, r = R_n) = 0 \text{ sur la surface latérale.}$$

⑩ Au total

$$\partial_t \rho - \frac{1}{n} \frac{\partial}{\partial n} \left(n \frac{\partial \rho}{\partial n} \right) = 0 \quad R_i < n < R_n$$

$$\rho(t=0, n) = \rho_0(n)$$

$$-\alpha \frac{\partial \rho}{\partial n}(t, n=R_i) = \Phi_+$$

$$-\alpha \frac{\partial \rho}{\partial n}(t, n=R_n) = 0$$

(11)

On retrouve le bilan global

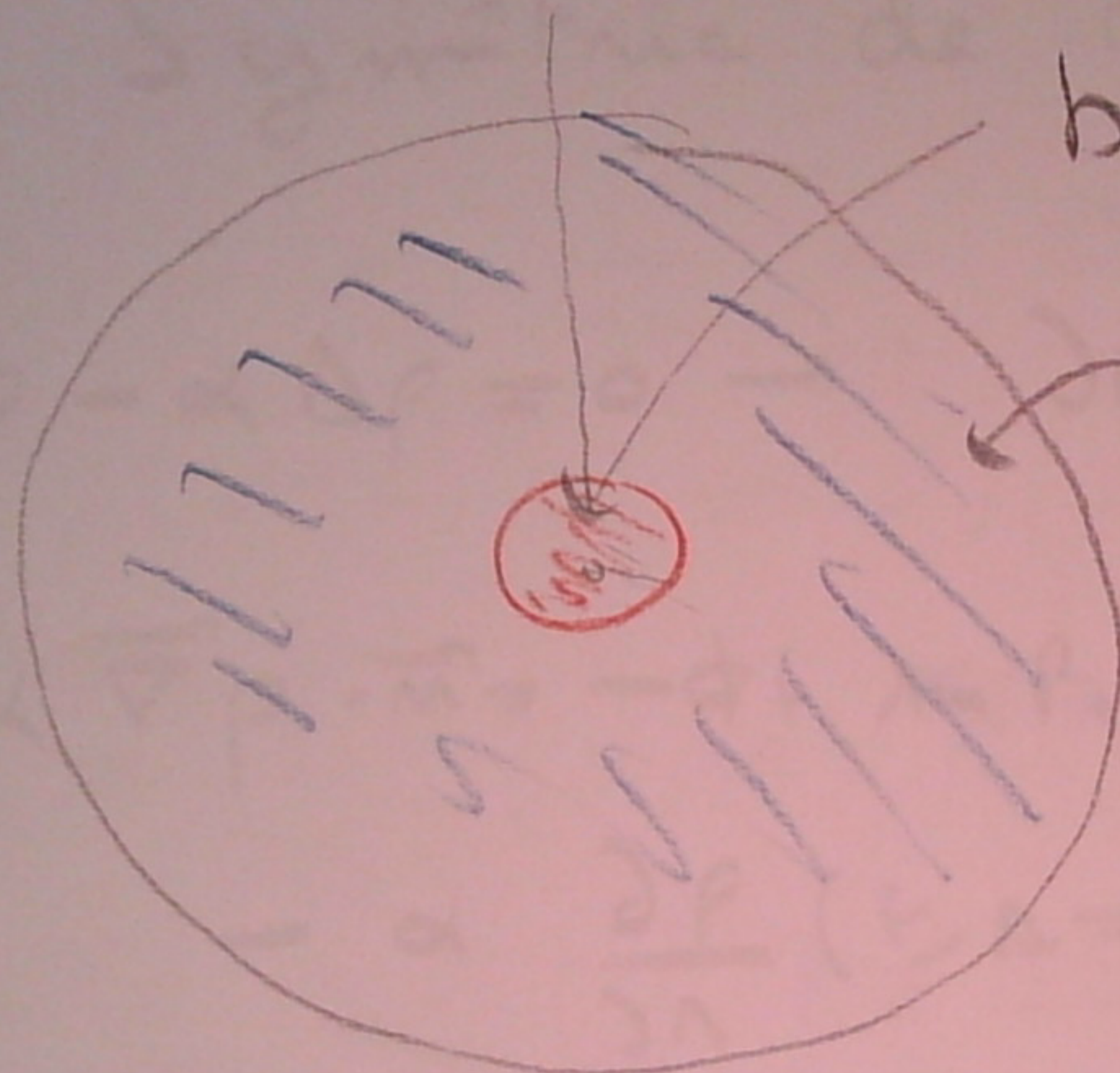
$$\int_{R_i}^{R_n} \left(\underbrace{\partial_r \rho}_{\downarrow} - \frac{\alpha}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \rho}{\partial r} \right) \right) r dr = 0$$

$$\begin{aligned} \rightarrow \frac{d}{dt} \int_{R_i}^{R_n} \rho r dr &= \int_{R_i}^{R_n} \alpha \frac{\partial}{\partial r} \left(r \frac{\partial \rho}{\partial r} \right) dr \\ &= \left[\alpha r \frac{\partial \rho}{\partial r} \right]_{R_i}^{R_n} \\ &= R_i \Phi_+ \end{aligned}$$

$$\text{et } Q = 2\pi L \int_{R_i}^{R_n} \rho r dr$$

$$\int \rho_+ dS = 2\pi L R_i \Phi_+$$

12



boule de sucre

Don

sphérique

$$D = \{ r \bar{e}_r, R_1 < r < R_2 \}$$

$$P_+ = \{ R_1 \bar{e}_r \} \quad \vec{n} = -\bar{e}_r$$

$$P_- = \{ R_2 \bar{e}_r \} \quad \vec{n} = +\bar{e}_r$$

$$\bar{e}_r = \cos\theta \bar{e}_3 + \sin\theta (a\psi \bar{e}_1 + b\psi \bar{e}_2)$$

Sphérique



(13)

Symétrie de notation

$$\begin{array}{|l} \rho(t, r, \theta, \varphi) = \rho(t, r) \\ \rho_0(r, \theta, \varphi) = \rho_0(r) \end{array}$$

$$\partial_t \rho - \alpha \Delta \rho = 0 \rightarrow \partial_t \rho - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \rho}{\partial r} \right) = 0$$

$$-\alpha \nabla \rho \cdot \vec{n} = -\phi_+ \quad \text{sur } P_+ \rightarrow$$

$$-\alpha \frac{\partial \rho}{\partial r} (t, r = R_i) = \phi_+$$

$$-\alpha \nabla \rho \cdot \vec{n} = 0 \quad \text{sur } P_0 \rightarrow$$

$$-\alpha \frac{\partial \rho}{\partial r} (t, r = R_n) = 0$$

(14) Au total

$$\partial_t \rho - \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \rho}{\partial r} \right) = 0 \quad R_i < r < R_n$$

$$\rho(t=0, r) = \rho_0(r)$$

$$- \alpha \frac{\partial \rho}{\partial r}(t, r=R_i) = \Phi_+$$

$$- \alpha \frac{\partial \rho}{\partial r}(t, r=R_n) = 0$$

(15) On retrouve le bilan global

$$\int_{R_i}^{R_n} \left(\frac{\partial p}{\partial t} - \frac{\phi}{n^2} \frac{\partial}{\partial n} \left(n^2 \frac{\partial p}{\partial n} \right) \right) n^2 dn = 0$$

$$\begin{aligned} \rightarrow \frac{d}{dt} \int_{R_i}^{R_n} p n^2 dn &= \int_{R_i}^{R_n} \alpha \frac{\partial}{\partial n} \left(n^2 \frac{\partial p}{\partial n} \right) dn \\ &= \left[\alpha n^2 \frac{\partial p}{\partial n} \right]_{R_i}^{R_n} \\ &= R_i^2 \phi_+ \end{aligned}$$

$$\text{et } Q = 4\pi \int_{R_i}^{R_n} p n^2 dn$$

$$\int p_+ dS = 4\pi R_i^2$$