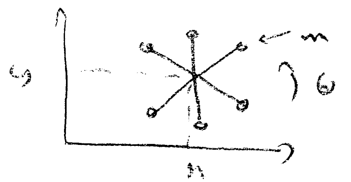
a) A priori les variables de configuration sont x, y, θ



b) $T_e = \frac{1}{2}(M+6m)(\dot{x}^2+\dot{y}^2) + 3mR^2\dot{\theta}^2$

on le sait parce que le hand-spinner est équilibré

Si on ne le sait pas, la position des masses est : pour $n = 0 \dots 5$

$\left(x + R\cos\left(\theta + n\frac{\pi}{3}\right)\right)\vec{e}_x + \left(y + R\sin\left(\theta + \frac{\pi}{3}\right)\right)\vec{e}_y$

la vitesse est alors

$\left(\dot{x} - R\dot{\theta}\sin\left(\theta + n\frac{\pi}{3}\right)\right)\vec{e}_x + \left(\dot{y} + R\dot{\theta}\cos\left(\theta + \frac{\pi}{3}\right)\right)\vec{e}_y$

et donc

$T_e = \frac{1}{2}M(\dot{x}^2+\dot{y}^2) + \frac{1}{2}\sum_{n=0}^{5} m\left(\left(\dot{x} - R\dot{\theta}\sin\left(\theta+n\frac{\pi}{3}\right)\right)^2\right.$
$\left.+ \left(\dot{y} + R\dot{\theta}\cos\left(\theta+\frac{\pi}{3}\right)\right)^2\right)$

$= \frac{1}{2}(M+6m)(\dot{x}^2+\dot{y}^2) + 3mR^2\dot{\theta}^2$
$+ m\sum_{n=0}^{5}\left(\underbrace{-\dot{x}R\dot{\theta}\sin\left(\theta+n\frac{\pi}{3}\right)}_{0} + \underbrace{\dot{y}R\dot{\theta}\cos\left(\theta+n\frac{\pi}{3}\right)}_{0}\right)$

d'où $L = \frac{1}{2}(M+6m)(\dot{x}^2+\dot{y}^2) + 3mR^2\dot{\theta}^2 + \mu B\cos\theta$

et
$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \rightarrow \ddot{x} = 0$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = \frac{\partial L}{\partial y} \rightarrow \ddot{y} = 0$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \theta} \rightarrow 6mR^2\ddot{\theta} = -\mu B\sin\theta$

Il n'y a pas de couplage entre les m^vt en x, y et le m^vt de rotation en θ.

c) si $\theta(t=0) = 0$ et $\dot{\theta}(t=0)$ alors $\dot{\theta}(t) = 0$

$\theta = 0$ est la position d'équilibre

d) pour de petits mouvements $\sin\theta \neq \theta$, et

$\ddot{\theta} + \frac{\mu B}{6mR^2}\theta = 0 \Rightarrow \omega = \sqrt{\frac{\mu B}{6mR^2}}$

e) idem a)

f) les masses sont $m_0 = m'$, $m_1 = m_2 = m_3 = m_4 = m_5 = m$

$T = \frac{1}{2}M(\dot{x}^2+\dot{y}^2) + \sum_{n=0}^{5}\left(\frac{1}{2}m_n\left(\left(\dot{x} - R\dot{\theta}\sin\left(\theta+n\frac{\pi}{3}\right)\right)^2\right.\right.$
$\left.\left.+ \left(\dot{y} + R\dot{\theta}\cos\left(\theta+n\frac{\pi}{3}\right)\right)^2\right)\right)$

$= T_e + \frac{1}{2}(m'-m)\left[(\dot{x} - R\dot{\theta}\sin\theta)^2 + (\dot{y} + R\dot{\theta}\cos\theta)^2\right]$

et toujours $L = T + W$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \rightarrow \frac{d}{dt}\Big((M+6m)\,\dot{x} + (m'-m)(\dot{x} - R\dot{\theta}\sin\theta)\Big) = 0 \rightarrow (M+5m+m')\,\ddot{x} + (m'-m)R\frac{d}{dt}(-\dot{\theta}\sin\theta)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} = \frac{\partial L}{\partial y} \rightarrow \frac{d}{dt}\Big((M+6m)\,\dot{y} + (m'-m)(\dot{y} + R\dot{\theta}\cos\theta)\Big) = 0 \rightarrow \text{———}\ \ddot{y} + \text{———} \frac{d}{dt}(\dot{\theta}\cos\theta)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \theta} \rightarrow \frac{d}{dt}\Big(6mR^2\dot{\theta} + (m'-m)\big[-R\sin\theta(\dot{x} - R\dot{\theta}\sin\theta) + R\cos\theta(\dot{y} + R\dot{\theta}\cos\theta)\big]\Big)$$

$$= -MB\sin\theta + (m'-m)\big[-(\dot{x} - R\dot{\theta}\sin\theta)R\dot{\theta}\cos\theta - (\dot{y} + R\dot{\theta}\cos\theta)R\dot{\theta}\sin\theta\big]$$

$$\frac{d}{dt}\Big((5m+m')R^2\dot{\theta} + (m'-m)R(\cos\theta\,\dot{y} - \sin\theta\,\dot{x})\Big) = -MB\sin\theta - (m'-m)(\dot{x}\cos\theta + \dot{y}\sin\theta)R\dot{\theta}$$

$$(5m+m')R^2\ddot{\theta} + MB\sin\theta + (m'-m)R(\cos\theta\,\ddot{y} - \sin\theta\,\ddot{x}) = 0$$

$$(5m+m')R^2\ddot{\theta} + MB\sin\theta + \frac{(m'-m)^2R^2}{M+5m+m'}\Big(-\cos\theta\frac{d}{dt}(\dot{\theta}\cos\theta) - \sin\theta\frac{d}{dt}(\dot{\theta}\sin\theta)\Big) = 0$$

$$\boxed{\left((5m+m') - \frac{(m'-m)^2}{M+5m+m'}\right)R^2\ddot{\theta} + MB\sin\theta = 0}$$

Pas demandé