a) $T = \frac{1}{2} m \begin{pmatrix} \dot{s} & \dot{\theta} \end{pmatrix} \begin{bmatrix} 2 & \frac{\cos(\alpha-\theta)\ell}{2} \\ \frac{\cos(\alpha-\theta)\ell}{2} & \ell^2 \end{bmatrix} \begin{pmatrix} \dot{s} \\ \dot{\theta} \end{pmatrix}$ ; $V = m g (2 \sin\alpha \, s - \cos\theta \, \ell)$

$L = T - V$

b) $\frac{d}{dt} \frac{\partial L}{\partial \dot{s}} = \frac{\partial L}{\partial s} \longrightarrow \frac{d}{dt}\left(m\left(2\dot{s} + \frac{\cos(\alpha-\theta)\ell}{2}\dot{\theta}\right)\right) = -2mg \sin\alpha$

$\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}} = \frac{\partial L}{\partial \theta} \longrightarrow \frac{d}{dt}\left(m\left(\frac{\cos(\alpha-\theta)\ell}{2}\dot{s} + \ell^2 \dot{\theta}\right)\right) = m \ell \sin(\alpha-\theta)\dot{s}\dot{\theta} - m g \ell \sin\theta$

c) $\begin{pmatrix} p_s \\ p_\theta \end{pmatrix} = m \begin{bmatrix} 2 & \frac{\cos(\alpha-\theta)\ell}{2} \\ \frac{\cos(\alpha-\theta)\ell}{2} & \ell^2 \end{bmatrix} \begin{pmatrix} \dot{s} \\ \dot{\theta} \end{pmatrix}$

d) $\begin{pmatrix} \dot{s} \\ \dot{\theta} \end{pmatrix} = \frac{1}{m\ell^2\left(2 - \frac{\cos(\alpha-\theta)^2}{2}\right)} \begin{bmatrix} \ell^2 & -\frac{\cos(\alpha-\theta)\ell}{2} \\ -\frac{\cos(\alpha-\theta)\ell}{2} & 2 \end{bmatrix} \begin{pmatrix} p_s \\ p_\theta \end{pmatrix}$

e) $\overline{T} = \begin{pmatrix} \dot{s} & \dot{\theta} \end{pmatrix} \begin{pmatrix} p_s \\ p_\theta \end{pmatrix} - T$ avec $\begin{pmatrix} \dot{s} \\ \dot{\theta} \end{pmatrix}$ défini par d)

$= \frac{1}{2} \frac{1}{m\ell^2\left(2 - \frac{\cos(\alpha-\theta)^2}{2}\right)} \begin{pmatrix} p_s & p_\theta \end{pmatrix} \begin{bmatrix} \ell^2 & -\frac{\cos(\alpha-\theta)\ell}{2} \\ -\frac{\cos(\alpha-\theta)\ell}{2} & 2 \end{bmatrix} \begin{pmatrix} p_s \\ p_\theta \end{pmatrix}$ & $H = \overline{T} + V$

f) $\frac{ds}{dt} = \frac{\partial H}{\partial p_s} \longrightarrow \frac{ds}{dt} = \frac{\left(\ell^2 p_s - \frac{\cos(\alpha-\theta)\ell}{2} p_\theta\right)}{m\ell^2\left(2 - \frac{\cos(\alpha-\theta)^2}{2}\right)}$ ; $\frac{d\theta}{dt} = \frac{\partial H}{\partial p_\theta} \longrightarrow \frac{d\theta}{dt} = \frac{\left(-\frac{\cos(\alpha-\theta)\ell}{2} p_s + 2 p_\theta\right)}{m\ell^2\left(2 - \frac{\cos(\alpha-\theta)^2}{2}\right)}$

$\frac{dp_s}{dt} = -\frac{\partial H}{\partial s} \longrightarrow \frac{dp_s}{dt} = -2 m g \sin\alpha$

$\frac{dp_\theta}{dt} = -\frac{\partial H}{\partial \theta} \longrightarrow \frac{dp_\theta}{dt} = \frac{-1}{2} \frac{\cos(\alpha-\theta)\sin(\alpha-\theta)}{\left(2 - \frac{\cos(\alpha-\theta)^2}{2}\right)^2} \begin{pmatrix} p_s & p_\theta \end{pmatrix} \begin{pmatrix} \ell^2 & -\frac{\cos(\alpha-\theta)\ell}{2} \\ -\frac{\cos(\alpha-\theta)\ell}{2} & 2 \end{pmatrix} \begin{pmatrix} p_s \\ p_\theta \end{pmatrix} + \frac{\sin(\alpha-\theta)\, p_s p_\theta}{2m\ell^2\left(2 - \frac{\cos(\alpha-\theta)^2}{2}\right)} - m g \ell \sin\theta$

là ok, c'était méchant