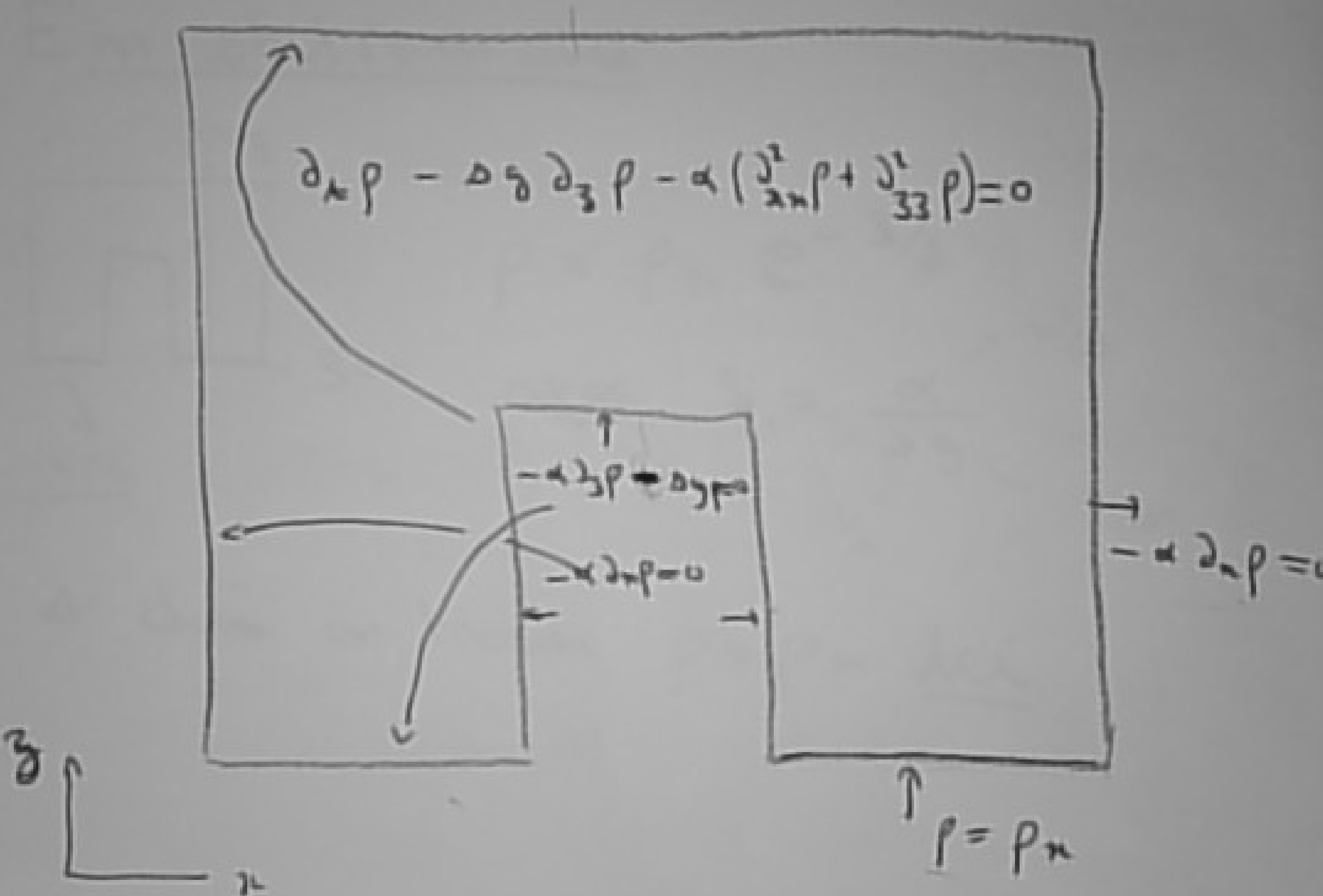
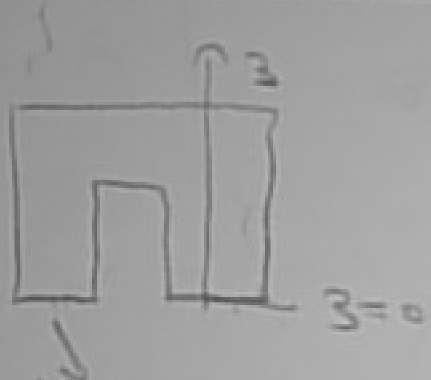

$\partial_x \rho - \Delta g \, \partial_z \rho - \alpha(\partial^2_{xx} \rho + \partial^2_{zz} \rho) = 0$
$-\alpha \partial_z \rho + \Delta g \rho = 0$
$-\alpha \partial_x \rho = 0$
$-\alpha \partial_x \rho = 0$
$\rho = \rho_x$
z
x

En stationnaire



ici

$$\rho = \rho_x \, e^{-z/\delta}$$

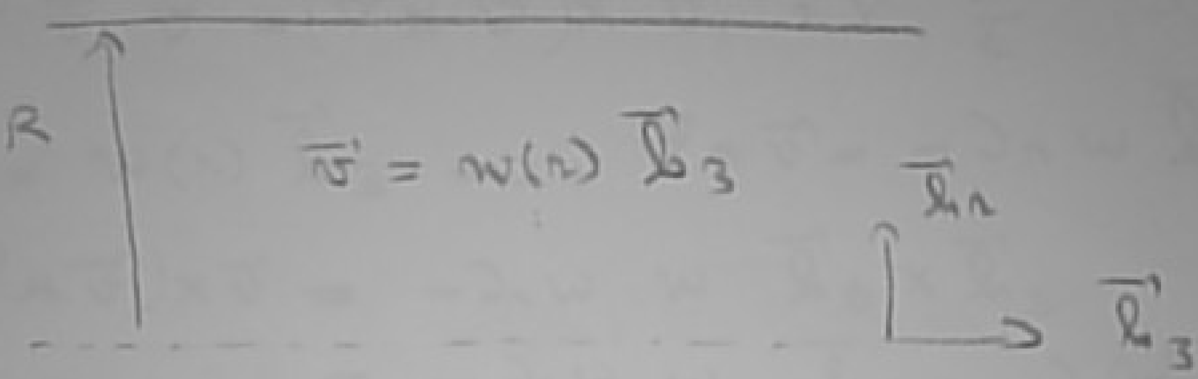
avec $\delta = \dfrac{\alpha}{s\, g}$

α : m^2/s

s : $1/s$

g : m/s^2

et donc on trouve $\rho = \rho_x$ **ici**



$$\begin{cases} \rho\left(\partial_t \vec{v} + \vec{\nabla}\vec{v}\;\vec{v}\right) + \vec{\nabla} p = -\eta\, \vec{\nabla}\times\vec{\nabla}\times\vec{v} \\ \underline{\vec{\nabla}\cdot\vec{v}} = 0 = \partial_3 w \end{cases}$$

assuré avec $\vec{v} = w(r)\,\vec{e}_3$

$$\vec{\nabla}_{\vec{v}}\,\vec{v} = (\vec{\nabla}\times\vec{v})\times\vec{v} + \vec{\nabla}\,\frac{\vec{v}^{2}}{2}$$

$$\vec{v} = \omega(r)\,\vec{e}_3 \implies \vec{\nabla}\times\vec{v} = -\partial_r\omega\,\vec{e}_\theta$$

$$\begin{aligned}(\vec{\nabla}\times\vec{v})\times\vec{v} &= -\partial_r\omega\;\omega\;\vec{e}_\theta\times\vec{e}_3\\ &= -\partial_r\omega\;\omega\;\vec{e}_r = -\partial_r\left(\frac{\omega^2}{2}\right)\vec{e}_r\end{aligned}$$

$$\vec{\nabla}\,\frac{\vec{v}^{2}}{2} = \partial_r\left(\frac{\omega^2}{2}\right)\vec{e}_r$$

$$\begin{aligned}\vec{\nabla}\times\vec{\nabla}\times\vec{v} &= \vec{\nabla}\times\left(-\partial_r\omega\,\vec{e}_\theta\right)\\ &= -\frac{1}{r}\,\partial_r\left(r\,\partial_r\omega\right)\vec{e}_3\end{aligned}$$

$$\vec{\nabla}\times\left(f(r)\,\vec{e}_\theta\right) = \frac{1}{r}\,\frac{\partial}{\partial r}(rf)\,\vec{e}_3$$

$$\partial_r P \, \vec{e}_r + \frac{1}{r} \partial_\theta P \, \vec{e}_\theta + \partial_z P \, \vec{e}_z = -\frac{\eta}{r} \partial_r (r \, \partial_r w) \, \vec{e}_z$$

soit

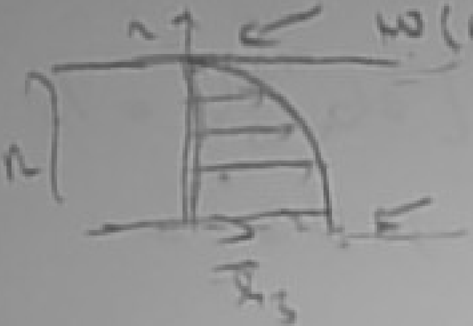
$$\left| \begin{array}{l} \partial_r P = 0 \\ \partial_\theta P = 0 \end{array} \right\} \Rightarrow P(r, \theta, z) = P(z)$$

et

$$\underbrace{\partial_z P}_{f^{ct} \text{ de } z} = \underbrace{-\frac{\eta}{r} \partial_r (r \, \partial_r w)}_{f^{ct} \text{ de } r}$$

Constante

$$\Rightarrow \left| \begin{array}{l} P = P_+ - \frac{\Delta P}{L} z \\ \frac{\eta}{r} \partial_r (r \, \partial_r w) = -\frac{\Delta P}{L} \end{array} \right.$$



$w(R)=0$ (c.l.)

$\partial_r w(0)=0$ (sinon $\vec{v}$ pas différentiable sur l'axe)

Eq sans 2° membre $\frac{1}{r}\partial_r\left(r\,\partial_r(w)\right)=0$.

$\longrightarrow$ 2 sols : $w=$ cst et ~~$w=\ln r$~~

d'où $w=\beta\,(R^2-r^2)$

avec $\frac{\eta}{r}\,\partial_r\left(-2\beta r^2\right)=-\Delta P/L$

$\longrightarrow \beta=\frac{\Delta P}{4\eta L}$

d'où $w=\frac{\Delta P}{4\eta L}\,(R^2-r^2)$

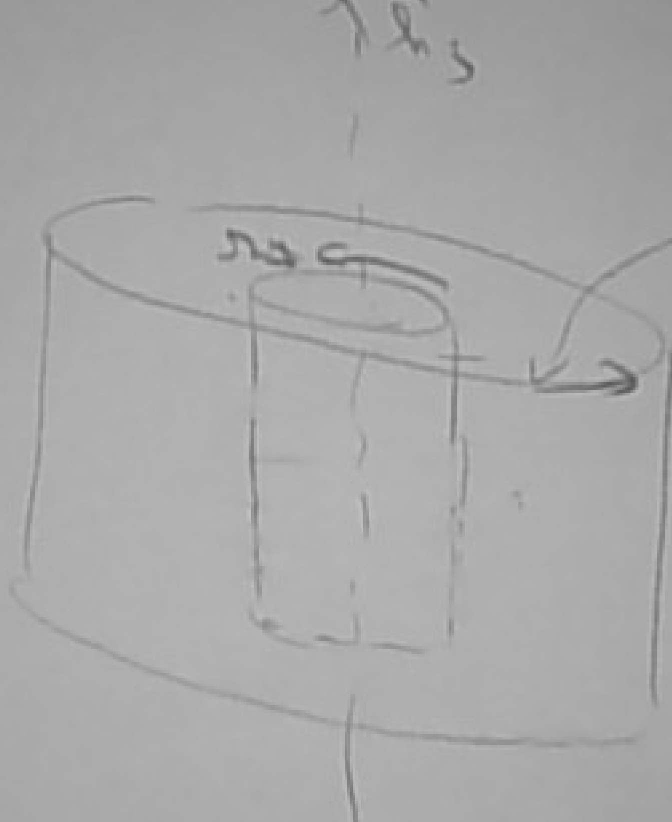
$$d = \int_0^{2\pi} d\theta \int_0^R r\,dr\, v \qquad (m^3/s)$$

$$= 2\pi \left(\frac{R^4}{2} - \frac{R^4}{4} \right) \frac{\Delta P}{4 \eta L}$$

$$\rightarrow \quad \Delta P = \frac{8 \eta L}{\pi R^4}\, d = \underline{\frac{128 \eta L}{\pi (2R)^4}\, d}$$

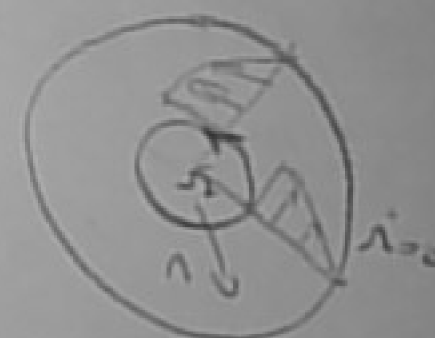
formule de perte de charge linéaire en régime laminaire.

Hélas c'est bien souvent en régime turbulent qu'on est → abaque de Moody



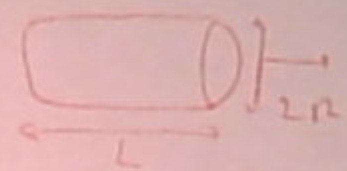
fluide

$\Omega_2 = 0$



$\vec{v} = v(r)\,\vec{e}_\theta$

$$\mathrm{Re} = \rho \frac{(2R)\,(d/\pi R^2)}{\eta}$$



$$= \rho \frac{R^3}{8\eta^2}\left(\frac{\Delta p}{L}\right) \qquad \frac{kg/m^3 \; m^3 \; Pa}{(Pa\,s)^2 \; m} \sim 1$$

Re < 2000 → ok pour la validité de la formule de pert. de charge

sinon pas valable