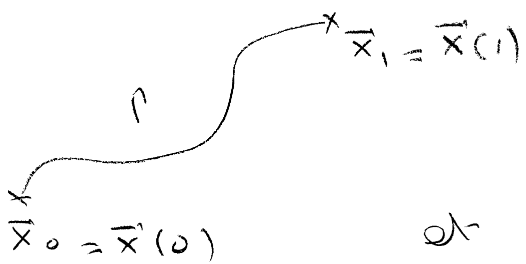
Gradient cylindrique

$$\vec{X}(s) = R(s)\,\vec{l}_{r}(\Theta(s)) + z(s)\,\vec{l}_3$$

$$\Rightarrow \vec{X}'(s) = R'\,\vec{l}_r + R\,\Theta'\,\vec{l}_\theta + z'\,\vec{l}_3$$



$$C = \int_\Gamma \vec{\nabla}\phi \cdot \vec{\tau}\, dl = \int_0^1 \vec{\nabla}\phi \cdot \vec{x}'(s)\, ds$$

on identifie et ça donne les composantes de $\vec{\nabla}\phi$

et on veut que $C = \phi(\vec{x}(1)) - \phi(\vec{x}(0))$

qui est égal à $= \int_0^1 \frac{d}{ds}\left(\phi(\vec{x}(s))\right) ds$

soit $\int_0^1 \left(\frac{\partial \phi}{\partial r} R' + \frac{\partial \phi}{\partial \theta} R\,\Theta' + \frac{\partial \phi}{\partial z} z' \right) ds$

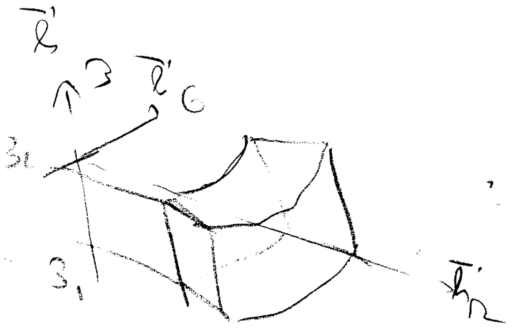
si $\vec{\nabla}\phi = \phi_r\,\vec{l}_r + \phi_\theta\,\vec{l}_\theta + \phi_3\,\vec{l}_3$

$$\int_0^1 \left(\left(\frac{\partial \phi}{\partial r} - \phi_r\right) R' + \left(R\frac{\partial \phi}{\partial \theta} - \phi_\theta\right) R\,\theta' + \left(\frac{\partial \phi}{\partial z} - \phi_3\right) z' \right) ds = 0$$

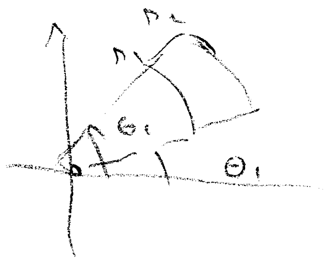
et cela $\forall\ R, \theta, z$ donc par exemple pour $R = R_0$ cste et $z = z_0$ cste

$\int_0^1 \left(R\frac{\partial \phi}{\partial \theta} - \phi_\theta\right) R_0\,\theta'\, ds = 0$ si $R\frac{\partial \phi}{\partial \theta} - \phi_\theta \neq 0$ on choisit $\theta' = R\frac{\partial \phi}{\partial \theta} - \phi_\theta$
et on arrive à une contradiction

Divergence cylindrique



$\vec{v} = v_r \vec{e}_r + v_\theta \vec{e}_\theta + v_z \vec{e}_z$



Par définition

$$\int_{z_1}^{z_2} dz \int_{r_1}^{r_2} dr \int_{\theta_1}^{\theta_2} r\, d\theta \; (\operatorname{div} \vec{v})\, dV$$

$$= \int_{r_1}^{r_2} dr \int_{\theta_1}^{\theta_2} r\, d\theta \left(v_z(r,\theta,z_2) - v_z(r,\theta,z_1) \right)$$

$$+ \int_{z_1}^{z_2} dz \int_{\theta_1}^{\theta_2} d\theta \left(r_2 v_r(r_2,\theta,z) - r_1 v_r(r_1,\theta,z) \right)$$

$$+ \int_{z_1}^{z_2} dz \int_{r_1}^{r_2} dr \left(v_\theta(r,\theta_2,z) - v_\theta(r,\theta_1,z) \right)$$

Si $z_1 = Z$, $r_1 = R$, $\theta_1 = \Theta$ et $z_2 = Z + \delta Z$, $r_2 = R + \delta R$, $\theta_2 = \Theta + \delta\Theta$

avec les grandeurs en δ petites, on fait un développ[ement] de Taylor pour obtenir

$$\delta Z\, \delta R\, R\, \delta\Theta\, (\operatorname{div} \vec{v})(R,\Theta,Z) = \begin{cases} R\,\delta R\, \delta\Theta \left(\frac{\partial v_z}{\partial z}(R,\Theta,Z)\, \delta Z \right) \\ + \delta Z\, \delta\Theta \left((R+\delta R)(v_r(R+\delta R,\Theta,Z)) - v_r(R,\Theta,Z) \right) \\ + \delta Z\, \delta R\, \frac{\partial v_\theta}{\partial \theta}(R,\Theta,Z) \end{cases}$$